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# MEMORANDUM

**TO:** Honorable Jeb Bush
Honorable Tom Gallagher
Honorable Charlie Crist

**FROM:** Coleman Stipanovich

**DATE:** February 13, 2003

**SUBJECT: Proposed Charges to the Lawton Chiles Endowment Fund Investment Plan**

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The following summarizes background information for SBA's staff recommendation and the process used to arrive at the proposed changes to the Lawton Chiles Endowment Fund Investment Plan. Back-up materials include this Memorandum, a slide presentation illustrating LCEF developments to date and key points from Dr. Moshe A. Milevsky's Consulting Report, the Consulting Report by Dr. Milevsky, Dr. Milevsky's Vita, and proposed revisions to the LCEF Investment Plan.

## A. Process

The principal work on this three-month study was conducted by Dr. Milevsky, who worked closely with SBA staff on the study during the LCEF Investment Plan review process.

The study was also reviewed and supported by our General Consultant, EnnisKnupp & Associates, and our Investment Consultant, Callan & Associates. At the end of the three-month study process, Dr. Milevsky and SBA staff conducted a one-day IAC workshop in December. Following this workshop, staff's recommendation to change the investment plan in order to preserve capital through a revised payout structure was unanimously supported by the IAC.

## B. Background

The Lawton Chiles Endowment Fund (LCEF) was created by act of the Legislature in 1999 to invest a portion of the tobacco settlement monies. The statute – section 215.5601, F.S. – sets forth the purpose of the endowment as follows:

> (a) Provide a perpetual source of enhanced funding for state children's health programs, child welfare programs, children's community-based health and human services initiatives, elder programs, and biomedical research activities related to tobacco use.
>
> (b) Use tobacco settlement monies as the source of enhanced funding to ensure the financial security of vital health and human services programs for children and elders.
>
> (c) Ensure that enhancement revenues will be available to help finance these important programs and initiatives.
>
> (d) Provide funds to help support public-health and biomedical research for the prevention, diagnosis, and treatment of diseases related to tobacco use.
>
> (e) Encourage the development of community-based solutions to strengthen and improve the quality of life of Florida's most vulnerable citizens, its children and elders. (e.s.)

The statute specifies that the Fund was to receive a total of $1.7 billion in contributions, according to the following schedule:

Table 1

| Fiscal Year | Appropriated Contribution |
|---|---|
| 1999-2000 | $1.1 billion |
| 2000-2001 | $200 million |
| 2001-2002 | $200 million |
| 2002-2003 | $200 million |

The last of these amounts was received in early January of this year. At this time, no further contributions are anticipated.

The State Board of Administration is required to administer the Fund and invest its assets "consistent with an investment plan developed by the executive director and approved by the board." (e.s.). Unlike the Florida Retirement System Trust Fund, there is no mechanism for regular annual contributions, no actuarial determination of funded status and no express liabilities or entitlements which represent claims on assets.

Instead, the statute sets forth an investment objective for the Fund and imposes certain requirements with respect to monies paid out from the Fund, to wit[1]:

> "The endowment shall be managed as an annuity. The investment objective shall be long-term preservation of the real value of the principal and a specified regular annual cash outflow for appropriation, as nonrecurring revenue."[2] (e.s.)

Preserving the value of the principal and, at the same time, making regular annual payouts are competing goals. In the extreme, the best way to preserve principal is by eliminating spending, and the best way to maintain spending (to a limit) is by eliminating (that is, using up) principal. The dual statutory objective of the LCEF is effectively tugging at two ends of the same rope. To establish an investment plan under this requirement, the SBA must implicitly or explicitly assign weightings to each goal.

Stated differently, one cannot successfully manage the Endowment to maintain a high likelihood of preserving principal while also making large annual payouts.

Similarly, the average size of the payouts and the degree of year-to-year consistency in payout amounts are competing goals. The more volatility one is willing to accept in year-to-year payout amounts, the higher can be the average payout over time. Table 2 summarizes these trade-offs and identifies the "policy levers" available to the SBA in managing the trade-offs.

Table 2

| **Competing Policy Objectives** (emphasizing any one requires compromising the other two) | **Key Investment Policy Decisions** (these decisions are made by the SBA based on the emphasis placed on each objective) |
|---|---|
| - High likelihood of preserving principle<br>- High payouts<br>- Fixed (or consistently growing) annual payout amounts | - Asset allocation<br>- Structure of annuity payout<br>- Target average payout amount<br>- Planning horizon |

The statute is quite clear with respect to preservation: the Endowment must be managed so that over the long-term, there is a high likelihood that net asset value of the Fund is equal to the nominal value of contributed capital (the principal) grown by the annual rate of inflation (real value). We believe that it is also clear that "regularity" of annual payouts does not necessarily mean that amounts must be fixed or consistently growing, since the payouts are characterized as "non-recurring."

[1] The law also imposes the following requirement. However, to date, it has not been implemented.

> "Notwithstanding s. 216.301 and pursuant to s. 216.351, all unencumbered balances of appropriations as of June 30 or undisbursed balances as of December 31 shall revert to the endowment's principal. Unencumbered or undisbursed balances appropriated for biomedical research shall revert to the principal in the separately reserved and accounted-for portion of the endowment established for biomedical research activities." (e.s.)

[2] 215.5601(4)(b), F.S. This section also specifies that a portion of the payout is earmarked solely for biomedical research activities and imposes a hold-back requirement thereon.

**The Current LCEF Total Fund Investment Plan (TFIP)**

A TFIP was initially adopted for the LCEF in mid-1999. That document established a fixed annuity payout schedule, which provided for an annual payout of $4.32, plus inflation per $100 of contributed capital.[3] It specified that the Fund's assets would be invested in U.S. equities, fixed income instruments and cash (short-term securities). The investment plan was modified late in 1999 to include allocations for international equities, Treasury Inflation-Protected Securities (TIPS) and real estate. SBA's staff and consultants expected the Fund's asset allocation to provide a long-term real return of 4.3% and a nominal return of just under 8%. At the time, consultants and staff noted that viability of the fixed annuity payout structure would be compromised if, in the initial years of the Fund, market returns were appreciably below long-term expectations.

Unfortunately, U.S. equity markets peaked in March of 2000, just after the final portion of the Fund's initial $1.1 billion funding was received.[4] From March 2000 through October 2002, U.S. and foreign equity markets lost approximately 50% of their value. Although the diversified nature of the Fund's portfolio has mitigated this loss somewhat, the Endowment's returns to date have nonetheless been disappointing, as illustrated in Table 3. Table 4 on the following page summarizes the valuation history of the Fund since inception.

Table 3

**Lawton Chiles Endowment Fund Return History**

| | Managed Return | Target Return | Value- Added |
|---|---|---|---|
| FY 1999-2000 | 8.91% | 8.41% | .50% |
| FY 2000-2001 | -8.16% | -7.97% | -.19% |
| FY 2001-2002 | -7.71% | -8.18% | .48% |
| 1st half FY 2002-2003 | -5.86% | -5.96% | .10% |

3 It also imposed the following caps on the amount of the payout, consistent with limitations in the statute as originally enacted:
For the appropriation available July 1, 2000: 3% of the fund net asset value on July 1, 1999;
For the appropriation available July 1, 2001: 4% of the fund average net asset value for the prior two years;
For the appropriation available July 1, 2002: 5% of the fund average net asset value for the prior three years; and
For appropriations available July 1, 2003 and each year thereafter: 6% of the fund average net asset value for the prior three years.

4 The Fund received $725.1 million in July 1999, $344.8 million in January 2000 and $30.1 million in February 2000.

Table 4

| Lawton Chiles Endowment Fund Valuation History | | | | |
|---|---|---|---|---|
| | Contributions | Distributions | Market Gain (Loss) | Ending Net Asset Value |
| | $m | $m | $m | $m |
| FY 1999-2000[a] | 1,100.0 | 0.0 | 81.0 | 1,181.0 |
| FY 2000-2001 | 200.0 | 27.4 | -96.8 | 1,256.8 |
| FY 2001-2002[b] | 200.0 | 55.7 | -108.4 | 1,292.7 |
| FY 2002-2003[c] thru Jan. 31 | 200.0 | 0.0 | -102.8 | 1,389.9 |
| Totals thru Jan. 31, 2003 | 1,700.0 | 83.1 | -227.0 | 1,389.9 |

[a] Opening balance July 1, 1999 was $725.1 million.

[b] Distribution amount includes $12.8m for 1st quarter of FY 2002-03.

[c] Total distribution for FY 2002-03 will be $51.5m.

Were the Fund to have maintained the real value of its principal to date, its net asset value (NAV) on January 31, 2003, would have been $1,805 million. Instead, at $1,389.9 million, the Fund's NAV is about $415 million or 23% less. While the preservation objective is a long-term goal (i.e. not one that can be met year in and year out), it is nonetheless troubling and significant that over a 3½-year period the Endowment has strayed so far from its target. Given the market-driven losses to date, there is less than a one in three chance that Endowment's principal will be preserved over a 30-year horizon. In the face of these losses, the odds are the Fund will not even survive for 30 years under the current spending policy, let alone retain real value.

## C. Proposed Revisions to the LCEF TFIP

In light of these developments, SBA staff initiated a study of the LCEF's investment plan in October of 2002. Dr. Moshe A. Milevsky, a professor of finance at York University in Toronto, who has successfully consulted to the Board in the past, was commissioned to work with Board staff and with SBA's general consultant in reexamining the Fund's asset allocation and annuity payout structure. Highlights of the study are as follows:

- **The current payout structure places excessive weight on regularity of payout, to the detriment of principal preservation.** Under the current TFIP, there is less than a one in three chance that the Endowment will meet its preservation objective (that is, have a net asset value in 30 years that is equal to the inflation-adjusted value of contributed capital).
- **The likelihood of attaining the preservation objective will not be materially enhanced by changing the Fund's asset allocation.** Even with a

100% equity allocation (which would provide the highest long-term expected return), the odds of meeting the objective would still be less than 50-50.

- **Although market risk could be reduced by increasing exposure to bonds, current payout rates could not be maintained.** Heavily weighting the Fund's asset allocation toward TIPS, a bond-like instrument particularly suitable to the Endowment, would not materially improve the likelihood of achieving the preservation objective *unless* payouts are cut significantly and the losses to date are ignored.[5]
- **Converting from a fixed to a participating annuity payout structure would dramatically increase the odds of preserving principal**. Under a participating annuity, the regular annual cash payout is adjusted, either positively or negatively, to reflect investment experience. If the LCEF adopts a strategy that pays x% on average, it has a much higher probability of preserving real value compared to a strategy that pays x% each and every year. The flexibility to adapt creates better odds.[6]
- **Both the Stanford and Harvard University endowments have a participating payout structure**. Although this approach introduces some year-to-year volatility into the amount paid out, total payouts over time will not necessarily decline. If, for example, markets recover and investment performance exceeds long-term averages, payouts would exceed those obtained through a fixed annuity structure.

There are a variety of ways in which a participating annuity payout structure could be formulated. The form recommended by SBA's staff and consultants follows the Stanford/Harvard model and consists of two terms: one representing the prior year payout and one representing the payout adjustment necessary to bring the Fund's principal back on track with the preservation objective. While the recommended payout formula is somewhat formidable when expressed in mathematical terms[7], its essential features are these:

1. To the extent earnings in any given year are greater than the long-term average expected return, payouts in subsequent years are adjusted upward, and vice versa.
2. The adjustment would be amortized over a number of years to dampen volatility in year-to-year payouts.
3. The formula has a term that explicitly maintains a specified probability of meeting the preservation objective. Staff recommends setting this parameter for an 80% probability.[8]

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[5] For example, investing 80% of the Endowment's asset in TIPS would necessitate cutting the payout rate by 1/3 and setting aside (ignoring) market losses to date in order to have a 90% chance of preserving the Endowment's remaining principle over 30 years.
[6] An intuitive analogy is this: You can lose more weight by eating only 3,000 calories on average per day, compared to eating 3,000 calories each and every day. You accomplish this by eating more after a vigorous day of activity and vice versa. The flexibility to spend $4.32 on average would create better preservation/survival odds than spending $4.32 each and every year.
[7] The specific formula appears in section VI of the proposed revisions to the LCEF Total Fund Investment Plan, draft 5.
[8] A higher probability lowers payouts in the face of any given shortage in principle, *ceteris paribus*.

4. The two terms of the formula, described above, are weighted to reflect the relative priority that is placed on payout stability vs. principle preservation. Staff recommends a 75%/25% respective weighting.[9]
5. The formula expressly adjusts for the planning horizon. Staff recommends a planning horizon of 30 years.[10]

For perspective, the following table summarizes the short-term fiscal impact of the staff recommendation compared to other alternatives. The total LCEF payout for the current fiscal year will be $51.5 million.

Table 5

| Alternative LCEF Investment Policies | FY 2003-04 Payout |
|---|---|
| 1. Staff recommendation: convert from a fixed to a participating annuity payout structure (parameters as indicated above) | $41.5 million |
| 2. Status quo: Maintain a fixed annual payout at $4.32 plus inflation per $100 of contributed capital, subject to a 6% cap[11] | $67.8 million |
| 3. Emphasize consistency in payouts: Make fixed annual payouts at the $4.32 inflation-adjusted rate | $78.0 million |

Also note, the Revenue Estimating Conference has recently adopted a LCEF payout estimate for FY 2003-04 of $41.0 million.[12]

## Conclusion

Given the market events that have transpired over the last three years, the payout structure adopted in the 1999 LCEF investment plan is untenable. There is less than a one in three chance that we will be able to preserve the real value of contributed capital over 30 years and continue to make the current level of fixed payouts. The level of payouts would have to be drastically lowered in order to continue making fixed payouts and have a high likelihood of preserving the principal.

Our study suggests it would be more prudent to allow the year-to-year payout amounts to be adjusted, within limits, based on market performance. Specifically, we suggest a participating annuity payout structure in which 75% weight would be given to year-to-year stability in payouts and 25% weight to investment experience. This approach, which is broadly consistent with what is done at Stanford, Harvard and other university

[9] These weights effectively control the term over which any given overage or underage in target vs. actual principle is amortized. Note, however, any weighting other than 0/100 reduces the probability of meeting the preservation objective.
[10] The shorter the planning horizon, the more annual payouts are dampened in the face of a principle shortfall, and vice versa.
[11] Per the current LCEF investment plan, payouts cannot exceed 6% of the Fund's average NAV over the prior 3 years.
[12] SBA staff recommends that the participating payout structure be made effective for the fiscal year that begins July 1, 2003.

endowments, will allow the average payout over time to be higher than under a fixed system, while greatly increasing the odds of preserving the real value of contributed principal over 30 years.

Please do not hesitate to contact me if you have any questions.

CS/db
cc: Kent Perez
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**MEMO**

To: Jim Francis and/or Kevin SigRist
Florida State Board of Administration

From: Moshe A. Milevsky
Executive Director

Date: December 10, 2002 (revised)

Subject: The Lawton Chiles Endowment Fund

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## 1 Executive Summary

The Lawton Chiles Endowment Fund (LCEF) has lost close to 20% of its real value since inception in late 1999. As of June 30th, 2002, the LCEF has $300 million less than the $1.59 billion balance needed to keep pace with inflation.

In the current environment, we estimate the probability that the value of the LCEF will exceed the inflation-adjusted value of the original contributions in the year 2029 – assuming current asset allocation and spending rules are maintained – is less than 35%. Even if the objective of the LCEF is modified (updated) to retain the *current* real purchasing power of the fund, the probability increases to only 55%.

We also find that tinkering with the current asset allocation – in bands of plus or minus 25% – will not do much to increase the probability of real sustainability (PoRS).

Therefore, our main actionable recommendation is to immediately reduce the current payout, and modify the operating rule to a more flexible spending policy that can dynamically adjust to changing market conditions. The proposed rule would be similar to strategies implemented by other large endowment funds. Implicitly, the strategy would create a 'reserve cushion' under which good years would subsidize bad ones.

While this action would increase the PoRS, it would obviously come at the expense of a sharp reduction in current spending. Nevertheless, we find that a policy that spends an average of $4.32 per $100 of invested principal during the next 30 years, has a $\frac{1}{3}$ to $\frac{1}{2}$ higher PoRS compared to a strategy that spends a fixed $4.32 each and every year.

In terms of asset allocation – given the wide range of estimates for the equity risk premium – we are somewhat ambivalent about incremental changes, since they will only marginally impact the PoRS. Our simulations indicate a bimodal outcome. Under the current rule, either the LCEF will beat the real benchmark by a wide margin, or it will fall dramatically short.

# 2 Maximizing Probability

The Total Fund Investment Plan (TFIP, Dec/3/1999) mentions two distinct investment objectives. The first is to maximize the probability of maintaining real value, and the second is to provide a specific real annual cash flow.

We believe that from a mathematical optimization point of view, the second so-called objective is essentially a constraint, while the probability maximization is the true objective function. This result is due to the fact that the Board can take actions (e.g. modify asset allocations) to increase and/or decrease the final estimated probability, but these same actions can not be taken to modify the second objective.

With that comment in mind, we believe that (a strict interpretation of) maximizing the probability of real sustainability (PoRS) can lead to some perverse outcomes which may be undesirable as a policy objective.

Note that maximizing the PoRS will result in excessively aggressive allocations when the fund is performing poorly and excessively conservative allocations when the fund performs well. These actions will however maximize the probability of sustainability. While such a counter-cyclical allocation might make sense, the magnitude required by strict adherence to the probability criteria can be excessive.

On a historical note, in the 1960's two well-known statisticians by the names of L. Dubins and J. Savage wrote a classic book entitled: "How to Gamble If You Must?" They argued that if all you care about is a 'financial goal' that you must reach within a certain amount of time, then you will be indifferent to the amount by which you beat the 'goal'. Furthermore, given the importance of reaching this goal, you are also indifferent about the amount by which you might shortfall the goal.

Dubins and Savage made the following qualitative insight by distinguishing between two cases. The first case is when a favorable game is available (i.e. you can reach the investment goal on average), and the second is when the game is unfavorable (i.e. given your current sum of money, and your expectation for earnings, you don't expect to reach the goal.) They derive the following rule: when your game is favorable, you let the law of large numbers help you, and bet small amounts to eventually reach your goal. But if the game is unfavorable, you should bet ALL your money at once, and hope for the best.

The investment insight from all of this is as follows: If LCEF truly wants to maximize the probability of beating the CPI benchmark, then we think there are three distinct financial regimes of interest. The first is when the fund is doing relatively well and the fund expects to achieve the 30 year goal. In that case, the LCEF should have a balanced asset allocation, *and the current allocation is as good as any.* However, if the value of the fund does very well – and the value of the fund gets close to the real present value the maturity target, the LCEF should move a substantial amount towards TIPS since that will help maximize the probability of reaching the target goal of preserving real value. Finally, if the fund is performing very poorly (like now), the fund is mathematically obligated to gamble its way out of the conundrum to maximize the chances of meeting the target. In this case, a mathematical algorithm would dictate 100% equity.

To put this in context, a recent paper in the *Journal of Portfolio Management* (Summer 1999) made the following statement. "*. . . The major problem with probability maximization is that the payoff function is binary valued 1 at the investment goal and 0 elsewhere. Therefore, if there is a finite deadline, significant risk-taking occurs near the deadline if wealth is far from the investment goal. . .*"

In sum, we recommend the objective function (or at least the wording) of the investment plan be modified to mitigate the undesirable impact of strictly adhering to this policy. Perhaps the objective can be stated as follows: *The Board will act to maximize the real spending rate of the LCEF subject to a constraint that the real value of the fund will exceed the original contribution with x% probability.* Then, depending on the pre-specified value of x% (dictated by the risk tolerance of the decision-makers) and the current value of the fund, the spending rate will emerge endogenously.

# 3 Understanding Long-term Returns

When discussing long-term economic assumptions, it is important to emphasize the critical distinction between the arithmetic mean (AM) and the geometric mean (GM) of the investment returns. While most practitioners refer to the AM when discussing market expectations, this number is less relevant for projecting long-term asset values. The geometric mean is the more relevant metric for long-term growth rates and it is a mathematical fact that the GM is always lower than the AM.

In periods of extraordinary volatility, the nominal expected return (i.e. arithmetic mean) from the stock market might be 10% per annum, but the expected value of $100 invested today, in ten years, might still be exactly $100 (which is a 0% growth rate). While this concept is counter-intuitive and paradoxical, it is at the heart of many (misplaced) debates on the magnitude of the equity premium.

We now state this in mathematical terms. The expected value of each dollar of initial investment, money or endowment fund in N years is:

$$(1 + GM)^N \tag{1}$$

As a rule of the thumb, the GM is obtained by subtracting half of the squared market volatility from the AM. Mathematically:

$$GM = AM - 0.5\sigma^2, \tag{2}$$

where the Greek symbol $\sigma$ denotes the (universal) measure of volatility. As one can see from the equation, if the volatility drag is high enough, the GM can actually go to zero.

Aside from understanding the equity premium, the GM is important from a spending point of view. The (new) rule we will advocate for the spending policy will be based on the projected Geometric Mean. More on this later.

# 4 The Equity Premium Literature

The performance of global equity markets during the last three years, has rekindled a spirited debate on the magnitude of the equity premium.

We define the equity premium as the difference between the expected return on the market portfolio and the risk-free interest rate. The existence of an equity premium has been explained as compensation investors receive for taking on the additional risk associated with equity investments. Investors receive compensation for taking on risk because most investors are economically "risk averse." Hence, the argument goes, to motivate such investors to invest in risky equity, they must be compensated with higher returns.

Technically, the term risk/return trade-off is somewhat misleading. In reality, the relationship between risk and return can be described as a "non-diversifiable risk/expected return" trade-off. This description takes into account the fact that only non-diversifiable risk is compensated with higher return. It also takes into account the obvious fact that there is no guarantee of superior returns for risky investment, just an expectation of superior returns. Therefore, in our opinion, as long as agregate stock markets remain volatile, investors can (continue to) expect to earn positive risk premiums over time.

The relationship between risk and return – which is the basis of many economic assumptions – is formalized in the Capital Asset Pricing Model (CAPM). The CAPM has been accepted by most finance academics, used by most practitioners in industry, and continues to be the main textbook treatment of the relationship between risk and return.

However, an influential (1992) academic paper by Eugene Fama and Kenneth French argued that beta cannot explain cross-sectional variations in asset returns. The Fama & French paper, as well as a number of others, have ignited a vigorous debate as to whether the CAPM actually represents an economic risk/return trade-off. Some have gone so far as to argue that beta has no explanatory power in predicting the cross variation in returns. This would imply that any global equity premium estimate that is predicated on a beta number, would be grossly inaccurate.

Nevertheless, historically, the nominal arithmetic mean (AM) annual return between the years 1926 and 2000 was 12.56%, with a standard deviation of 19.67%. This is equivalent to a geometric mean (GM) annual return of 10.7%.

Despite the strong theoretical and empirical arguments in favor of the existence of an equity premium, there exists a wide range of estimates regarding the forward-looking magnitude. There is a wide dispersion of opinions, and we now review the most prominent ones. We start with the many pessimists and then progress to the few optimists.

In another article, Fama and French (2001) present evidence regarding the U.S. equity premium by comparing expectations which are based on dividends and earnings growth models, against observed equity premiums. The find that equity premium expectations based on dividends and earnings growth models were similar to realized equity premiums between the years 1872-1950, with a value of approximately 4.25%. However, between the years 1951-2000, the estimated equity premiums based on the dividends and earnings growth models were 2.55% and 4.32% respectively. This is much below the observed equity premium of 7.43% during the same period.

Fama and French argue that future expectations should be formed based on the dividends and earnings growth models, and not based on observed historical average returns. They note that stock prices have grown faster than dividends. They (prophetically) suggest that stock returns will decrease, to align equity premiums to dividend growth.

They go on to provide three explanations for the expected decline in equity premiums: (1) More individuals and institutions participate in the markets than before. This argument receives support from Heaton and Lucas (2000), who estimate that recent increases in stock market participation have resulted in a 2% decline in the equity premium. (2) Diversified equity portfolios are cheaply accessible, through mutual funds. This argument receives support from Merton (1987), who argues that incomplete diversification can result in significantly higher equity premiums. (3) The stock market may be mean reverting.

Fama and French find the mean-reversion argument particularly compelling. Mean reversion may be due to macroeconomic factors, as argued by Fama and French (1989), or irrational investor behavior, as argued by Shiller (1989). Regardless, Fama and French are unequivocal that investors face a period of low expected returns. The practical implication of the Fama and French (2001) research is that U.S. equities should form a (much) smaller fraction of any well diversified portfolio.

Following on the same theme, Jagannathan et al. (2000) presents empirical evidence to argue that the U.S. equity premium has declined since the early 1970s. They contend that while the estimated equity premium averaged 7% between 1926 through 1970, it averaged only 0.7% between 1970 and 1999, and 0% between 1982 and 1999.

While Jagannathan et al. (2000) do not provide a definitive explanation for the equity premium decline, they rationalize their empirical findings by arguing that the equity premium is a function of two dynamics: (1) The riskiness associated with stocks, relative to other investments, such as risk-free treasury securities. Investors require an equity premium for taking on these risks. (2) Market imperfections, such as transaction costs and imperfect information. These imperfections decrease investor willingness to invest in stocks, and therefore result in higher stock premiums.

Jagannathan et al. argue that their observation of low equity premiums may be due to an easing of market imperfections since the 1970s. They argue that market imperfections have eased due to technological improvements over the past few decades. Further, since these technological improvements are expected to endure, the equity premium will remain at the observed level indefinitely.

Claus and Thomas (1999) claim that traditional estimates of equity premiums are too high for the 1985-1999 period. They base their argument by estimating the discount rate that equates current prices to forecasts of dividends and earnings, using dividend growth-style model. They find that dividend growth rates of 12% are required to support an equity premium of 7%. They argue that a 12% dividend growth rate is unrealistic, and therefore argue that an equity premium of 7% is equally unrealistic. They rationalize historical evidence that suggests the equity premium is higher than their estimates by arguing that the historical equity premium is a statistical aberration, or due to survivorship bias.

Arnott and Bernstein (2002) continue along the same (pessimistic) theme and argue that observed real stock returns over the past 75 years were extraordinary, and unlikely to be repeated. Therefore, they argue, extrapolating these extraordinary results to the future is dangerous. Arnott and Bernstein conclude that the equity risk premium should be closer to 2.4%.

It is (slightly) more difficult to locate academic optimists when it comes to the magnitude of the equity premium.

Interestingly, a survey conducted by Welch (2000) consisting of finance and economics academics indicates that generally they do not expect equity premiums to decline greatly. Welch (2000) surveyed 226 professors, and found their long-horizon estimate of the equity premium to be 7%. Expectations were not greatly diverse: the standard deviation associated with the estimates was only 2%. The shorter-term expectations – over a one-year horizon - were 5.8% on average, with a larger standard deviation of 4.5%.

In 2001, Welch updated the previous year's study by surveying 510 finance and economics professors. The 2001 study found that their long-horizon estimate of the equity premium ranged between 5% and 5.5%, while their short-term expectation ranged between 3% and 3.5%. Welch attributes this decline to pessimistic academic papers, such as Fama and French (2001). Hence, while academics do not expect equity premiums to decline greatly, they have become more pessimistic over the past few years.

Chen and Ibbotson (2002) are more optimistic about long-term equity premiums. They predict premiums only slightly below historical averages. They argue that long-term equity premiums are 3.97% on a geometric basis and 5.9% on an arithmetic basis. Chen and Ibbotson derive these figures by using a supply-side approach. Specifically, they decompose returns into a number of market and economic factors. These factors include book value, dividends, the dividend payout ratio, earnings, GDP, inflation, the P/E ratio, and ROE.

The lesson we take from this and other literature is as follows:

- There is strong consensus that an equity premium does, in fact, exist. This implies that a well-diversified portfolio of U.S. and international equities can be expected to perform better than a portfolio of fixed income securities over the very long run. In fact, the longer the time horizon, the greater the probability of outperformance.
- However, there is a contentious debate regarding the forward-looking magnitude of this premium. One can locate intelligent arguments in favour of an equity premium – or spread over bonds – in the range of 1% to 5%. Therefore, given the heightened sensitivity of portfolio optimizers to the expected return from the various (input) asset classes, it is virtually impossible to make a forceful argument in favour of any prespecified equity/bonds mix, *even* if we hold risk-preference constant.
- Nevertheless, even the most optimistic of equity premium pundits project forward-looking returns that are lower than historical values. Many of them describe the last 20 years of U.S. capital market history as a 'lucky' draw from a fair distribution. In the fact of such evidence, it is very hard to justify increasing the fund's equity exposure.

# 5 The Spending Rule

The spending rule I am proposing is a weighted average of two separate sub-rules. The first sub-rule corresponds to the current LCEF policy – which spends a fixed real dollar-valued amount subject to a cap – while the second sub-rule is related to a fraction of the current market value of the endowment. The weighting rule can be expressed in the following algebraic manner:

$$s_i = (1-\lambda)s_{i-1} + \lambda V_i \max[GM - \frac{K+R}{T}, 0], \tag{3}$$

where $s_i$ is the real (dollar value) spending at time period (or year) $i$, $V_i$ is the real fund value at time $i$, $GM$ is the anticipated real growth rate (a.k.a. geometric mean return) of the fund, $K$ is a prudence constant (which I will soon explain, but initially can be taken as $K = 1$), $T$ is the terminal horizon over which the fund would like to fulfil the mandate of retaining real value, and $R := \ln[V_0/V_i]$ is defined equal to the total return required to bring the fund back to it's original real value. Without any loss of generality, we can normalize the original time-zero value of the fund to be $V_0 = \$100$, in which case $R := \ln[100/V_i]$, which can then be thought-of as a percentage. Finally, if we let the symbol $\pi_i$ denote the

CPI level at time period $i$, $\pi_0 = 1$ and $\pi_i/\pi_{i-1}$ is the inflation rate, then we can convert the real value of the fund and/or spending amount to current nominal values, by multiplying into $V_i\pi_i$ and $s_i\pi_i$.

The functional form listed in equation (3) is a variant of the so-called Stanford University endowment rule, which has also been adopted by Yale University and a variety of other endowment funds. The value of $\lambda$ is usually between 20% and 40%, which implicitly weighs recent consumption more heavily. However, in the case of Stanford and Yale, $V_i$ is listed as the *nominal average value of the fund during the previous 3-5 years*, and the second sub-rule is (simply) $\lambda V_i c$, where the constant $c$ is a fixed target long-term payout rate on the order of 4.5% to 5.5%. Of course, if an endowment fund spends 5% of its (average) real value in any given year, it will also be spending 5% of its (average) nominal value in that year, since the inflation term is multiplicative.

The reason we have adopted a more complex expression $\max[GM - (K+R)/T, 0]$, instead of a fixed constant $c$ for the spending rate, is because of the LCEF's desire to control the Probability of Real Sustainability (PoRS) at some terminal horizon $T$. Indeed, if $T = \infty$, the rule collapses to $\lambda V_i GM$, where $GM$ is the anticipated geometric mean, which would likely be in the vicinity of 4.5% to 5.5%.

Note, also, that as time marches on, we are not advocating that $T$ be reduced each year, by one year. Rather, our opinion is that a fixed $T$ be used for the life of the fund in a rolling manner. More on this later.

# 6 Examples.

For example if we use a weighting factor of $\lambda = 1$, and the original value of the fund was $V_0 = \$100$, while the current value (at time $i$) of the fund is $V_i = 80$, the fund would require an instantaneous return of $R = \ln[100/80] = 0.223$, or 22.3% in real terms, to recoup losses. Therefore, if the anticipated geometric mean (a.k.a. median) return is 5.5%, and the terminal horizon is $T = 30$, the real spending rule listed in equation (3), would imply:

$$s_i = \$80 \max[0.055 - \frac{1 + 0.223}{30}, 0] = \$1.138, \tag{4}$$

which is much less than (the old) *\$4.32 per \$100* rule, mainly because the fund is showing a 22.3% real loss since inception. If, however, we adopt a $\lambda = 40\%$ weighting, and assuming previous spending was $s_{i-1} = \$4.32$, we obtain:

$$s_i = (1 - 0.4)4.32 + (0.4)80 \max[0.055 - \frac{1 + 0.223}{30}, 0] = \$3.047. \tag{5}$$

Back to the $\lambda = 1$ case, if our terminal horizon were to increase to $T = 50$ years from $T = 30$ years, the real spending rule would be:

$$s_i = \$80 \max[0.055 - \frac{1 + 0.223}{50}, 0] = \$2.443, \tag{6}$$

which is obviously more than \$1.138 (but still less than \$4.32) because the terminal horizon – which is the point at which the fund must recuperate – has been extended.

Heuristically, we are amortizing any losses over an extended period of time, and we are spending investment gains – above and beyond the geometric mean return – over the same period. This, *de facto*, creates a reserve within the endowment fund which is then used to subsidize losses in bad periods.

Finally, if the terminal horizon is reduced to $T = 15$ years, and $\lambda = 1$, the real spending rule would be reduced to:

$$s_i = \$80 \max[0.055 - \frac{1 + 0.223}{15}, 0] = \$0. \quad (7)$$

Yes, the spending would be reduced to zero. This is because the fund can not afford to spend, and still achieve the financial objective within 15 years with a reasonable (to be discussed) probability. Of course, using the weighted average version of the spending rule, by assuming that $\lambda = 0.40$ for example, then only 40% weight would be placed on the above-mentioned zero spending, and the other 60% would be determined by the previous year's consumption. In fact, we are leaning towards recommending such a (stabilizing) 40/60 weighting, at least for the first few years of the transition to the new rule.

In the other direction, if the fund were to achieve a real value of $V_i = 130$ at time period $i$, the $R$ value would be $R = \ln[100/130] = -0.262$, which implies that the fund could afford to lose 26.2% and still retain the original mandate. Naturally, it also implies the fund can afford to spend more in this period, and we obtain:

$$s_i = \$130 \max[0.055 - \frac{1 - 0.262}{30}, 0] = \$3.952, \quad (8)$$

in real terms, when the terminal horizon is $T = 30$ years and $\lambda = 1$.

Some additional comparative statics are as follows: As we mentioned earlier, when $T = \infty$, and the endowment fund has a truly *infinite* horizon, the spending policy under a $\lambda = 1$ weighting collapses to $V_i GM$, which is a fixed fraction of the current real value of the fund. In our earlier $GM = 0.055$ case, this would imply spending 5.5% of the fund value in each and every year. Note that under a finite ($T < \infty$) horizon, real spending is usually constrained to be less than 5.5% of fund value. This should be evident from equation (3), where $(1 + R)/T$ is likely to be positive, since $R$ is likely to be greater than $-1$. (But, if the prudence constant is much less than 1, and the fund value is much higher than its original value, it *is* possible the spending rate will exceed the geometric mean parameter.)

## 7 Understanding Prudence

The 'prudence' constant $K$, which we initial took to be valued at one is directly related to the probability of real sustainability (PoRS) at the terminal horizon $T$. The larger the value of the prudence constant $K$, the greater is the probability. In fact, I will later 'prove' that if we adopt the weighting $\lambda = 1$ version of the Stanford/Yale endowment spending rule (i.e. with second term $\lambda V_i c$) then the exact PoRS can be calculated in continuous time. Fitting $c$ to our problem by setting it equal to $\max[GM - (K+R)/T, 0]$ gives:

$$\text{PoRS} = \Phi\left(\frac{K}{\sigma\sqrt{T}}\right), \tag{9}$$

assuming log-normal investment returns, where $\sigma$ is the projected volatility (standard deviation) of the endowment fund and $\Phi(a)$ represents the Cumulative Distribution Function (CDF) of the standard normal variate, evaluated at $a$. Thus, for example, if $\sigma = 20\%$, $T = 30$ and $K = 1$, we get:

$$\text{PoRS} = \Phi\left(\frac{1}{(0.20)(5.477)}\right) = 0.82 = 82\% \tag{10}$$

The probability of real sustainability at the terminal horizon $T$, by adopting an $s_i = V_i(GM - (1+R)/T)$ rule is exactly 82%. Since the real spending rule we are advocating will actually vary over time, unlike the Stanford/Yale rule, equation (9) is only an approximation to the true PoRS. But this approximation appears to be reasonable.

As a general rule of the thumb, when the projected volatility of the endowment fund's real return is less than 20% and the terminal horizon is at least 30 years, then a spending rule that uses a prudence constant of $K = 1$ in equation (3), will result in a probability of real sustainability (PoRS) that is greater than 80%. But, if we use half the prudence value, so that $K = 0.5$, the implied PoRS from equation (9) becomes 67% (under a $\sigma = 20\%$), and spending will be higher as well. Indeed, in contrast to equation (4), real spending would be:

$$s_i = \$80 \max[0.055 - \frac{0.5 + 0.223}{30}, 0] = \$2.472 \tag{11}$$

when the fund has lost 20% of real value. Contrast this with a real spending of \$1.138, when $K = 1$. Intuitively, decreasing prudence $K$, is equivalent to increasing the terminal horizon $T$, over which the gains and losses are spread. If we let the prudence constant $K = 0$, so that spending becomes: $V_i(GM - R/T)$, the PoRS in equation (9) will collapse to $\Phi(0) = 0.5$, and the probability of sustainability is exactly 50%.

It is important to note that our probability statement only holds true if the spending rate is positive. In other words, assuming $GM - (K+R)/T$ is greater than zero, so that spending is positive, the PoRS will be determined by equation (9). But, for example, if the fund has performed horrendously, so that $V_i << V_0$, the spending 'tap' will be extinguished and the probability of real sustainability will be less than $\left(K/\sigma\sqrt{T}\right)$.

The attached spreadsheet output displays the impact and effect of $\lambda$ on the spending policy, for the (1970-2000) historic sequence of investment returns and inflation, assuming a 60% allocation to domestic equity, and a 40% allocation to fixed income. (These are based on Ibbotson Associates numbers. The attached spreadsheet allows for a general asset allocation.) The last three charts display the distributing of spending during the next 30 years, assuming the current asset allocation is maintained.

The difference between the respective charts, is the value of $\lambda$. In the first (and fourth) chart, $\lambda = 1$ which places zero weighting on previous consumption. The second (and fifth) chart takes $\lambda = 0.5$ which places 50% weighting on previous consumption and the third (and sixth) chart takes $\lambda = 0$, which places 80% weight on previous consumption. Notice that as $\lambda$ declines the rule generates a 'smoother' spending policy. In fact, at the extreme, when $\lambda = 0$ and 100% weighting is placed on previous consumption, the spending rule would collapse to a fixed \$4.32 per \$100 which was the previous rule (excluding the cap.)

# 8 Appendix: The Proof.

Assume the real value of the LCEF obeys the following diffusion process:

$$dV_t = (\mu - s^*)V_t dt + \sigma V_t dB_t, \qquad V_0 = 100 \tag{12}$$

where $\mu$ denotes the real (infinitesimal) expected return, $\sigma$ denotes the real volatility and $s^*$ denotes a fixed spending rate. The solution to the SDE in equation (12), can be represented as:

$$V_{t+T} = V_t \exp\{(\mu - s^* - 0.5\sigma^2)T + \sigma B_T\}, \tag{13}$$

where $V_t$ denotes the time-$t$ real value of the fund. This is the classical log-normal assumption for wealth, since $\ln[V_{t+T}/V_t]$ is normal distributed with a mean value $(\mu - s^* - 0.5\sigma^2)T$, and a standard deviation of $\sigma\sqrt{T}$. Note, also, that in any given year mean value of the underlying log-normal distribution is $\exp\{\mu\}$, and the median (a.k.a. geometric mean) value is precisely $\exp\{(\mu - 0.5\sigma^2)\}$. The spending-rate term $s^*$ can be viewed as a dividend yield that is continuously withdrawn from the fund. The actual real dollar value withdrawn, would be $s^*V_t$, which corresponds with the earlier $s_i$ term.

Using this notation and framework, the probability of real sustainability (PoRS), can be expressed as:

$$\text{PoRS} := \Pr\left[V_{t+T} \geq V_0\right], \tag{14}$$

which leads to:

$$\begin{aligned} \text{PoRS} &= \Pr\left[V_t \exp\{(\mu - s^* - 0.5\sigma^2)T + \sigma B_T\} \geq 100\right] \\ &= \Pr\left[(\mu - s^* - 0.5\sigma^2)T + \sigma B_T \geq \ln\left(\frac{100}{V_t}\right)\right] \\ &= \Pr\left[\mathbf{Z} \geq \frac{\ln\left(\frac{V_0}{V_t}\right) - (\mu - s^* - 0.5\sigma^2)T}{\sigma\sqrt{T}}\right], \end{aligned} \tag{15}$$

where $\mathbf{Z}$ denotes a standard-normal random variable. Now, if we let $GM$ denote the (log) median annual return, $GM = \mu - 0.5\sigma^2$, and we let $R$ denote the total return required to obtain the original real value $R = \ln[V_0/V_t]$, we have:

$$\text{PoRS} = 1 - \Phi\left(\frac{R - (GM - s^*)T}{\sigma\sqrt{T}}\right). \tag{16}$$

Finally, if we substitute the spending rate implicit in equation (3):

$$s^* = GM - \frac{K + R}{T},$$

into equation (16), we are left with:

$$\begin{aligned} \text{PoRS} &= 1 - \Phi\left(\frac{R - (GM - GM + \frac{K+R}{T})T}{\sigma\sqrt{T}}\right) \\ &= 1 - \Phi\left(\frac{-K}{\sigma\sqrt{T}}\right) = \Phi\left(\frac{K}{\sigma\sqrt{T}}\right), \end{aligned} \tag{17}$$

with the last equality coming from the symmetry of the normal distribution. This is exactly the expression given in equation (9). Q.E.D.

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encl: 6 charts, 2 spreadsheets

$$s_i = (1-\lambda)s_{i-1} + (\lambda)V_i \max[\, GM - \frac{R+K}{30}, 0]$$

| **Input Region:** | |
|---|---|
| Real Expected Return: | 5.9% |
| Standard Deviation | 13.0% |
| Terminal Horizon: | 30 |
| Shortfall Tolerance: | 25% |
| Lambda [0,1]: | 1 |
| Initial Spending: | $4.32 |
| | |
| Real Geometric Mean: | 5.06% |
| Prudence (K) | 0.48112 |
| | |
| Average Consumption: | $ 3.81 |
| | |
| Ending Fund Value: | $ 172.0 |

| **Year** | **Real Return** | **Inflation Rate** | (e.o.y) **Nominal Fund** | **CPI** | (e.o.y) **Real Fund** | **R** | **Real Spending** | **Nominal Spending** |
|---|---|---|---|---|---|---|---|---|
| 1970 | 1.6% | 5.49% | $ 107.16 | 1.055 | $ 101.59 | -1.6% | $ 3.562 | $ 3.757 |
| 1971 | 9.7% | 3.36% | $ 117.24 | 1.090 | $ 107.52 | -7.3% | $ 3.973 | $ 4.332 |
| 1972 | 9.3% | 3.41% | $ 127.60 | 1.128 | $ 113.17 | -12.4% | $ 4.375 | $ 4.933 |
| 1973 | -18.4% | 8.80% | $ 108.92 | 1.227 | $ 88.78 | 11.9% | $ 2.714 | $ 3.329 |
| 1974 | -28.3% | 12.20% | $ 84.99 | 1.376 | $ 61.75 | 48.2% | $ 1.140 | $ 1.569 |
| 1975 | 15.7% | 7.01% | $ 103.31 | 1.473 | $ 70.14 | 35.5% | $ 1.593 | $ 2.346 |
| 1976 | 14.3% | 4.81% | $ 120.97 | 1.544 | $ 78.36 | 24.4% | $ 2.069 | $ 3.195 |
| 1977 | -11.3% | 6.77% | $ 111.55 | 1.648 | $ 67.68 | 39.0% | $ 1.456 | $ 2.400 |
| 1978 | -5.3% | 9.03% | $ 112.69 | 1.797 | $ 62.70 | 46.7% | $ 1.190 | $ 2.138 |
| 1979 | -2.8% | 13.31% | $ 121.71 | 2.036 | $ 59.77 | 51.5% | $ 1.039 | $ 2.115 |
| 1980 | 3.5% | 12.40% | $ 139.19 | 2.289 | $ 60.81 | 49.7% | $ 1.092 | $ 2.499 |
| 1981 | -10.8% | 8.94% | $ 132.76 | 2.493 | $ 53.24 | 63.0% | $ 0.720 | $ 1.796 |
| 1982 | 21.4% | 3.87% | $ 165.15 | 2.590 | $ 63.77 | 45.0% | $ 1.246 | $ 3.226 |
| 1983 | 8.7% | 3.80% | $ 182.72 | 2.688 | $ 67.97 | 38.6% | $ 1.472 | $ 3.958 |
| 1984 | 5.5% | 3.95% | $ 196.10 | 2.795 | $ 70.17 | 35.4% | $ 1.595 | $ 4.457 |
| 1985 | 23.8% | 3.77% | $ 246.24 | 2.900 | $ 84.92 | 16.4% | $ 2.470 | $ 7.162 |
| 1986 | 17.8% | 1.13% | $ 284.87 | 2.933 | $ 97.14 | 2.9% | $ 3.261 | $ 9.562 |
| 1987 | -2.4% | 4.41% | $ 280.68 | 3.062 | $ 91.67 | 8.7% | $ 2.900 | $ 8.879 |
| 1988 | 8.7% | 4.42% | $ 308.47 | 3.197 | $ 96.48 | 3.6% | $ 3.217 | $ 10.284 |
| 1989 | 18.5% | 4.65% | $ 369.91 | 3.346 | $ 110.55 | -10.0% | $ 4.188 | $ 14.011 |
| 1990 | -5.5% | 6.11% | $ 357.00 | 3.550 | $ 100.55 | -0.6% | $ 3.491 | $ 12.394 |
| 1991 | 20.0% | 3.06% | $ 426.32 | 3.659 | $ 116.51 | -15.3% | $ 4.617 | $ 16.895 |
| 1992 | 4.7% | 2.90% | $ 440.99 | 3.765 | $ 117.12 | -15.8% | $ 4.662 | $ 17.552 |
| 1993 | 9.7% | 2.75% | $ 477.29 | 3.869 | $ 123.37 | -21.0% | $ 5.124 | $ 19.825 |
| 1994 | -5.1% | 2.67% | $ 445.78 | 3.972 | $ 112.23 | -11.5% | $ 4.307 | $ 17.109 |
| 1995 | 27.6% | 2.54% | $ 560.76 | 4.073 | $ 137.68 | -32.0% | $ 6.222 | $ 25.342 |
| 1996 | 8.8% | 3.32% | $ 601.96 | 4.208 | $ 143.05 | -35.8% | $ 6.647 | $ 27.971 |
| 1997 | 21.3% | 1.92% | $ 709.36 | 4.289 | $ 165.39 | -50.3% | $ 8.486 | $ 36.394 |
| 1998 | 18.4% | 1.61% | $ 809.58 | 4.358 | $ 185.77 | -61.9% | $ 10.251 | $ 44.672 |
| 1999 | 5.1% | 2.68% | $ 825.13 | 4.475 | $ 184.40 | -61.2% | $ 10.129 | $ 45.326 |
| 2000 | -1.3% | 3.39% | $ 795.91 | 4.626 | $ 172.03 | -54.3% | $ 9.052 | $ 41.879 |

$$s_i = (1-\lambda)s_{i-1} + (\lambda)V_i \max[\,GM - \frac{R+K}{30}, 0]$$

| **Input Region:** | |
|---|---|
| Real Expected Return: | 5.9% |
| Standard Deviation | 13.0% |
| Terminal Horizon: | 30 |
| Shortfall Tolerance: | 25% |
| Lambda [0,1]: | 0.5 |
| Initial Spending: | $4.32 |
| | |
| Real Geometric Mean: | 5.06% |
| Prudence (K) | 0.48112 |
| | |
| Average Consumption: | $ 3.63 |
| | |
| Ending Fund Value: | $ 178.0 |

| **Year** | **Real Return** | **Inflation Rate** | (e.o.y) **Nominal Fund** | **CPI** | (e.o.y) **Real Fund** | **R** | **Real Spending** | **Nominal Spending** |
|---|---|---|---|---|---|---|---|---|
| 1970 | 1.6% | 5.49% | $ 107.16 | 1.055 | $ 101.59 | -1.6% | $ 3.941 | $ 4.157 |
| 1971 | 9.7% | 3.36% | $ 116.78 | 1.090 | $ 107.11 | -6.9% | $ 3.942 | $ 4.299 |
| 1972 | 9.3% | 3.41% | $ 127.13 | 1.128 | $ 112.75 | -12.0% | $ 4.143 | $ 4.672 |
| 1973 | -18.4% | 8.80% | $ 108.73 | 1.227 | $ 88.63 | 12.1% | $ 3.424 | $ 4.200 |
| 1974 | -28.3% | 12.20% | $ 84.14 | 1.376 | $ 61.13 | 49.2% | $ 2.266 | $ 3.119 |
| 1975 | 15.7% | 7.01% | $ 100.33 | 1.473 | $ 68.12 | 38.4% | $ 1.873 | $ 2.759 |
| 1976 | 14.3% | 4.81% | $ 116.91 | 1.544 | $ 75.73 | 27.8% | $ 1.894 | $ 2.923 |
| 1977 | -11.3% | 6.77% | $ 107.96 | 1.648 | $ 65.50 | 42.3% | $ 1.616 | $ 2.663 |
| 1978 | -5.3% | 9.03% | $ 108.71 | 1.797 | $ 60.49 | 50.3% | $ 1.346 | $ 2.418 |
| 1979 | -2.8% | 13.31% | $ 117.02 | 2.036 | $ 57.47 | 55.4% | $ 1.135 | $ 2.310 |
| 1980 | 3.5% | 12.40% | $ 133.51 | 2.289 | $ 58.33 | 53.9% | $ 1.050 | $ 2.404 |
| 1981 | -10.8% | 8.94% | $ 127.33 | 2.493 | $ 51.07 | 67.2% | $ 0.835 | $ 2.082 |
| 1982 | 21.4% | 3.87% | $ 157.95 | 2.590 | $ 60.99 | 49.5% | $ 0.968 | $ 2.507 |
| 1983 | 8.7% | 3.80% | $ 175.40 | 2.688 | $ 65.25 | 42.7% | $ 1.146 | $ 3.081 |
| 1984 | 5.5% | 3.95% | $ 189.04 | 2.795 | $ 67.65 | 39.1% | $ 1.300 | $ 3.634 |
| 1985 | 23.8% | 3.77% | $ 238.22 | 2.900 | $ 82.15 | 19.7% | $ 1.799 | $ 5.218 |
| 1986 | 17.8% | 1.13% | $ 277.63 | 2.933 | $ 94.67 | 5.5% | $ 2.448 | $ 7.179 |
| 1987 | -2.4% | 4.41% | $ 275.72 | 3.062 | $ 90.05 | 10.5% | $ 2.622 | $ 8.027 |
| 1988 | 8.7% | 4.42% | $ 303.82 | 3.197 | $ 95.02 | 5.1% | $ 2.871 | $ 9.179 |
| 1989 | 18.5% | 4.65% | $ 365.50 | 3.346 | $ 109.24 | -8.8% | $ 3.482 | $ 11.652 |
| 1990 | -5.5% | 6.11% | $ 354.95 | 3.550 | $ 99.98 | 0.0% | $ 3.467 | $ 12.310 |
| 1991 | 20.0% | 3.06% | $ 423.90 | 3.659 | $ 115.85 | -14.7% | $ 4.018 | $ 14.702 |
| 1992 | 4.7% | 2.90% | $ 440.73 | 3.765 | $ 117.06 | -15.7% | $ 4.337 | $ 16.331 |
| 1993 | 9.7% | 2.75% | $ 478.38 | 3.869 | $ 123.65 | -21.2% | $ 4.741 | $ 18.343 |
| 1994 | -5.1% | 2.67% | $ 448.28 | 3.972 | $ 112.86 | -12.1% | $ 4.547 | $ 18.061 |
| 1995 | 27.6% | 2.54% | $ 562.79 | 4.073 | $ 138.18 | -32.3% | $ 5.404 | $ 22.011 |
| 1996 | 8.8% | 3.32% | $ 607.99 | 4.208 | $ 144.48 | -36.8% | $ 6.083 | $ 25.598 |
| 1997 | 21.3% | 1.92% | $ 719.74 | 4.289 | $ 167.82 | -51.8% | $ 7.387 | $ 31.682 |
| 1998 | 18.4% | 1.61% | $ 827.75 | 4.358 | $ 189.94 | -64.2% | $ 9.004 | $ 39.240 |
| 1999 | 5.1% | 2.68% | $ 850.59 | 4.475 | $ 190.09 | -64.2% | $ 9.819 | $ 43.938 |
| 2000 | -1.3% | 3.39% | $ 823.30 | 4.626 | $ 177.96 | -57.6% | $ 9.692 | $ 44.839 |

$$s_i = (1-\lambda)s_{i-1} + (\lambda)V_i \max[\,GM - \frac{R+K}{30}, 0]$$

**Input Region:**

| | |
|---|---|
| Real Expected Return: | 5.9% |
| Standard Deviation | 13.0% |
| Terminal Horizon: | 30 |
| Shortfall Tolerance: | 25% |
| Lambda [0,1]: | 0.2 |
| Initial Spending: | $4.32 |
| Real Geometric Mean: | 5.06% |
| Prudence (K) | 0.48112 |
| Average Consumption: | $ 3.06 |
| Ending Fund Value: | $ 182.2 |

| Year | Real Return | Inflation Rate | (e.o.y) Nominal Fund | CPI | (e.o.y) Real Fund | R | Real Spending | Nominal Spending |
|---|---|---|---|---|---|---|---|---|
| 1970 | 1.6% | 5.49% | $ 107.16 | 1.055 | $ 101.59 | -1.6% | $ 4.168 | $ 4.397 |
| 1971 | 9.7% | 3.36% | $ 116.51 | 1.090 | $ 106.86 | -6.6% | $ 4.120 | $ 4.492 |
| 1972 | 9.3% | 3.41% | $ 126.60 | 1.128 | $ 112.28 | -11.6% | $ 4.158 | $ 4.688 |
| 1973 | -18.4% | 8.80% | $ 108.24 | 1.227 | $ 88.24 | 12.5% | $ 3.862 | $ 4.738 |
| 1974 | -28.3% | 12.20% | $ 83.32 | 1.376 | $ 60.53 | 50.2% | $ 3.305 | $ 4.550 |
| 1975 | 15.7% | 7.01% | $ 97.54 | 1.473 | $ 66.22 | 41.2% | $ 2.920 | $ 4.300 |
| 1976 | 14.3% | 4.81% | $ 111.73 | 1.544 | $ 72.37 | 32.3% | $ 2.680 | $ 4.137 |
| 1977 | -11.3% | 6.77% | $ 101.89 | 1.648 | $ 61.82 | 48.1% | $ 2.372 | $ 3.910 |
| 1978 | -5.3% | 9.03% | $ 101.16 | 1.797 | $ 56.29 | 57.5% | $ 2.071 | $ 3.722 |
| 1979 | -2.8% | 13.31% | $ 107.28 | 2.036 | $ 52.68 | 64.1% | $ 1.796 | $ 3.657 |
| 1980 | 3.5% | 12.40% | $ 120.60 | 2.289 | $ 52.69 | 64.1% | $ 1.575 | $ 3.606 |
| 1981 | -10.8% | 8.94% | $ 113.63 | 2.493 | $ 45.57 | 78.6% | $ 1.336 | $ 3.332 |
| 1982 | 21.4% | 3.87% | $ 139.09 | 2.590 | $ 53.70 | 62.2% | $ 1.217 | $ 3.153 |
| 1983 | 8.7% | 3.80% | $ 153.39 | 2.688 | $ 57.06 | 56.1% | $ 1.155 | $ 3.104 |
| 1984 | 5.5% | 3.95% | $ 164.87 | 2.795 | $ 59.00 | 52.8% | $ 1.124 | $ 3.140 |
| 1985 | 23.8% | 3.77% | $ 207.80 | 2.900 | $ 71.66 | 33.3% | $ 1.235 | $ 3.580 |
| 1986 | 17.8% | 1.13% | $ 243.34 | 2.933 | $ 82.98 | 18.7% | $ 1.458 | $ 4.274 |
| 1987 | -2.4% | 4.41% | $ 243.72 | 3.062 | $ 79.60 | 22.8% | $ 1.595 | $ 4.883 |
| 1988 | 8.7% | 4.42% | $ 271.07 | 3.197 | $ 84.78 | 16.5% | $ 1.768 | $ 5.653 |
| 1989 | 18.5% | 4.65% | $ 329.25 | 3.346 | $ 98.40 | 1.6% | $ 2.084 | $ 6.971 |
| 1990 | -5.5% | 6.11% | $ 323.28 | 3.550 | $ 91.06 | 9.4% | $ 2.239 | $ 7.949 |
| 1991 | 20.0% | 3.06% | $ 390.11 | 3.659 | $ 106.62 | -6.4% | $ 2.573 | $ 9.415 |
| 1992 | 4.7% | 2.90% | $ 410.04 | 3.765 | $ 108.90 | -8.5% | $ 2.872 | $ 10.815 |
| 1993 | 9.7% | 2.75% | $ 450.00 | 3.869 | $ 116.32 | -15.1% | $ 3.219 | $ 12.452 |
| 1994 | -5.1% | 2.67% | $ 426.37 | 3.972 | $ 107.34 | -7.1% | $ 3.367 | $ 13.374 |
| 1995 | 27.6% | 2.54% | $ 540.25 | 4.073 | $ 132.65 | -28.3% | $ 3.860 | $ 15.720 |
| 1996 | 8.8% | 3.32% | $ 589.72 | 4.208 | $ 140.14 | -33.7% | $ 4.371 | $ 18.393 |
| 1997 | 21.3% | 1.92% | $ 706.07 | 4.289 | $ 164.63 | -49.9% | $ 5.181 | $ 22.220 |
| 1998 | 18.4% | 1.61% | $ 822.69 | 4.358 | $ 188.78 | -63.5% | $ 6.248 | $ 27.229 |
| 1999 | 5.1% | 2.68% | $ 858.08 | 4.475 | $ 191.76 | -65.1% | $ 7.155 | $ 32.019 |
| 2000 | -1.3% | 3.39% | $ 843.12 | 4.626 | $ 182.24 | -60.0% | $ 7.712 | $ 35.680 |

# Monte Carlo Simulation Analysis of Managed Endowment Spending Program (MESP)

Asset Allocation

| Wilshire | Lehman | MSCIxUS | TIPS | Cash | REIT |
|---|---|---|---|---|---|
| 56.00% | 16.00% | 12.00% | 11.00% | 1.00% | 4.00% |

**Input Region:**

| | |
|---|---|
| Real Expected Return: | 7.5% |
| Standard Deviation | 20.0% |
| Terminal Horizon: | 30 |
| Shortfall Tolerance: | 18% |
| Lambda [0,1]: | 1 |
| Initial Spending: | $4.32 |
| Initial Wealth | $100 |
| Current Value | $100 |

$$s_i = (1-\lambda)s_{i-1} + \lambda V_i \max[GM - \frac{Ln(100/V_i)+K}{T}, 0]$$

| | |
|---|---|
| Real Geometric Mean: | 5.50% |
| Prudence (K) | 1.00273 |

| | |
|---|---|
| **Prob. of Maintaining $100 (after inflation) at 30-year Horizon** | 85.89% |

**Output in Real Value**

*The percentile value of actual real spending -- in each year -- based on the given rule.*

| Year | 5% | 10% | 20% | 30% | 40% | 50% | 60% | 70% | 80% | 90% | 95% |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2001 | 1.48 | 1.64 | 1.85 | 2.01 | 2.17 | 2.31 | 2.46 | 2.63 | 2.83 | 3.15 | 3.44 |
| 2002 | 1.26 | 1.51 | 1.78 | 2.02 | 2.23 | 2.45 | 2.68 | 2.95 | 3.26 | 3.72 | 4.18 |
| 2003 | 1.15 | 1.42 | 1.77 | 2.05 | 2.32 | 2.59 | 2.88 | 3.24 | 3.66 | 4.29 | 4.85 |
| 2004 | 1.06 | 1.36 | 1.78 | 2.11 | 2.41 | 2.74 | 3.07 | 3.50 | 4.00 | 4.80 | 5.53 |
| 2005 | 0.99 | 1.34 | 1.79 | 2.17 | 2.52 | 2.89 | 3.28 | 3.73 | 4.36 | 5.26 | 6.14 |
| 2006 | 0.95 | 1.32 | 1.83 | 2.26 | 2.64 | 3.04 | 3.49 | 4.01 | 4.70 | 5.81 | 6.89 |
| 2007 | 0.93 | 1.31 | 1.84 | 2.31 | 2.75 | 3.21 | 3.69 | 4.28 | 5.04 | 6.29 | 7.49 |
| 2008 | 0.90 | 1.33 | 1.91 | 2.40 | 2.89 | 3.37 | 3.91 | 4.56 | 5.41 | 6.79 | 8.12 |
| 2009 | 0.88 | 1.31 | 1.93 | 2.50 | 3.02 | 3.57 | 4.13 | 4.83 | 5.76 | 7.26 | 8.67 |
| 2010 | 0.83 | 1.29 | 1.99 | 2.58 | 3.12 | 3.69 | 4.28 | 5.07 | 6.09 | 7.71 | 9.23 |
| 2011 | 0.80 | 1.30 | 2.00 | 2.60 | 3.20 | 3.81 | 4.55 | 5.37 | 6.44 | 8.14 | 9.76 |
| 2012 | 0.79 | 1.30 | 2.05 | 2.68 | 3.31 | 3.98 | 4.72 | 5.58 | 6.75 | 8.65 | 10.41 |
| 2013 | 0.77 | 1.29 | 2.11 | 2.79 | 3.43 | 4.12 | 4.91 | 5.86 | 7.09 | 9.17 | 11.21 |
| 2014 | 0.80 | 1.35 | 2.13 | 2.85 | 3.51 | 4.26 | 5.05 | 6.06 | 7.37 | 9.53 | 11.85 |
| 2015 | 0.79 | 1.33 | 2.20 | 2.90 | 3.61 | 4.38 | 5.22 | 6.33 | 7.77 | 10.14 | 12.30 |
| 2016 | 0.77 | 1.36 | 2.22 | 2.99 | 3.74 | 4.52 | 5.40 | 6.55 | 8.07 | 10.46 | 12.78 |
| 2017 | 0.82 | 1.41 | 2.30 | 3.07 | 3.86 | 4.66 | 5.60 | 6.75 | 8.30 | 10.91 | 13.36 |
| 2018 | 0.82 | 1.44 | 2.29 | 3.10 | 3.91 | 4.79 | 5.77 | 6.95 | 8.56 | 11.33 | 14.15 |
| 2019 | 0.86 | 1.45 | 2.36 | 3.19 | 4.02 | 4.91 | 5.92 | 7.18 | 8.80 | 11.63 | 14.67 |
| 2020 | 0.84 | 1.45 | 2.38 | 3.24 | 4.06 | 4.98 | 6.04 | 7.28 | 9.04 | 11.99 | 15.03 |
| 2021 | 0.84 | 1.50 | 2.42 | 3.30 | 4.15 | 5.17 | 6.24 | 7.49 | 9.32 | 12.57 | 15.64 |
| 2022 | 0.84 | 1.52 | 2.50 | 3.38 | 4.28 | 5.23 | 6.30 | 7.69 | 9.58 | 12.93 | 16.06 |
| 2023 | 0.87 | 1.57 | 2.54 | 3.43 | 4.34 | 5.33 | 6.52 | 7.90 | 9.90 | 13.19 | 16.31 |
| 2024 | 0.89 | 1.58 | 2.60 | 3.49 | 4.47 | 5.51 | 6.64 | 8.15 | 10.17 | 13.47 | 16.86 |
| 2025 | 0.93 | 1.62 | 2.67 | 3.60 | 4.56 | 5.60 | 6.78 | 8.34 | 10.43 | 13.83 | 17.47 |
| 2026 | 0.90 | 1.63 | 2.69 | 3.68 | 4.64 | 5.73 | 6.93 | 8.47 | 10.63 | 14.20 | 17.53 |
| 2027 | 0.90 | 1.67 | 2.75 | 3.72 | 4.73 | 5.83 | 7.08 | 8.62 | 10.90 | 14.51 | 17.85 |
| 2028 | 0.98 | 1.67 | 2.83 | 3.81 | 4.84 | 5.94 | 7.20 | 8.73 | 10.98 | 14.76 | 18.60 |
| 2029 | 0.96 | 1.71 | 2.85 | 3.92 | 4.91 | 6.03 | 7.36 | 9.05 | 11.27 | 15.05 | 18.82 |
| 2030 | 0.99 | 1.78 | 2.91 | 3.95 | 4.94 | 6.11 | 7.50 | 9.16 | 11.48 | 15.35 | 19.24 |
| **LCEF Real Maturity Value** | $ 76.02 | $ 91.19 | $ 110.26 | $ 126.13 | $ 140.31 | $ 156.08 | $ 173.72 | $ 193.73 | $ 220.16 | $ 261.24 | $ 300.04 |

# Monte Carlo Simulation Analysis of Managed Endowment Spending Program (MESP)

Asset Allocation

| Wilshire | Lehman | MSCIxUS | TIPS | Cash | REIT |
|---|---|---|---|---|---|
| 56.00% | 16.00% | 12.00% | 11.00% | 1.00% | 4.00% |

**Input Region:**

| | |
|---|---|
| Real Expected Return: | 7.5% |
| Standard Deviation | 20.0% |
| Terminal Horizon: | 30 |
| Shortfall Tolerance: | 18% |
| Lambda [0,1]: | 0.5 |
| Initial Spending: | $4.32 |
| Initial Wealth | $100 |
| Current Value | $100 |

$$s_i = (1-\lambda)s_{i-1} + \lambda V_i \max[GM - \frac{Ln(100/V_i)+K}{T}, 0]$$

| | |
|---|---|
| Real Geometric Mean: | 5.50% |
| Prudence (K) | 1.00273 |

| | |
|---|---|
| **Prob. of Maintaining $100 (after inflation) at 30-year Horizon** | 84.76% |

**Output in Real Value**

*The percentile value of actual real spending -- in each year -- based on the given rule.*

| Year | 5% | 10% | 20% | 30% | 40% | 50% | 60% | 70% | 80% | 90% | 95% |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2001 | 2.90 | 2.98 | 3.09 | 3.17 | 3.24 | 3.32 | 3.39 | 3.48 | 3.58 | 3.74 | 3.88 |
| 2002 | 2.10 | 2.26 | 2.44 | 2.58 | 2.72 | 2.86 | 3.01 | 3.17 | 3.36 | 3.66 | 3.93 |
| 2003 | 1.65 | 1.84 | 2.10 | 2.30 | 2.49 | 2.69 | 2.91 | 3.14 | 3.44 | 3.88 | 4.29 |
| 2004 | 1.37 | 1.60 | 1.92 | 2.18 | 2.42 | 2.68 | 2.95 | 3.26 | 3.65 | 4.26 | 4.80 |
| 2005 | 1.19 | 1.45 | 1.85 | 2.15 | 2.45 | 2.74 | 3.05 | 3.44 | 3.93 | 4.66 | 5.37 |
| 2006 | 1.07 | 1.38 | 1.82 | 2.18 | 2.51 | 2.85 | 3.23 | 3.68 | 4.26 | 5.18 | 5.98 |
| 2007 | 0.98 | 1.34 | 1.81 | 2.22 | 2.62 | 3.00 | 3.42 | 3.93 | 4.61 | 5.67 | 6.68 |
| 2008 | 0.92 | 1.32 | 1.85 | 2.29 | 2.72 | 3.17 | 3.65 | 4.20 | 4.97 | 6.21 | 7.36 |
| 2009 | 0.90 | 1.31 | 1.88 | 2.37 | 2.86 | 3.33 | 3.87 | 4.49 | 5.35 | 6.72 | 7.96 |
| 2010 | 0.87 | 1.31 | 1.93 | 2.47 | 2.98 | 3.49 | 4.07 | 4.78 | 5.68 | 7.26 | 8.62 |
| 2011 | 0.82 | 1.31 | 1.96 | 2.52 | 3.09 | 3.66 | 4.29 | 5.05 | 6.09 | 7.69 | 9.25 |
| 2012 | 0.80 | 1.31 | 1.99 | 2.60 | 3.19 | 3.81 | 4.51 | 5.34 | 6.38 | 8.18 | 9.90 |
| 2013 | 0.80 | 1.28 | 2.04 | 2.69 | 3.29 | 3.97 | 4.69 | 5.61 | 6.75 | 8.70 | 10.61 |
| 2014 | 0.80 | 1.30 | 2.08 | 2.76 | 3.41 | 4.10 | 4.88 | 5.85 | 7.09 | 9.24 | 11.35 |
| 2015 | 0.78 | 1.32 | 2.13 | 2.84 | 3.54 | 4.25 | 5.08 | 6.13 | 7.47 | 9.74 | 11.99 |
| 2016 | 0.76 | 1.33 | 2.19 | 2.91 | 3.63 | 4.39 | 5.29 | 6.38 | 7.86 | 10.29 | 12.56 |
| 2017 | 0.78 | 1.37 | 2.23 | 3.01 | 3.74 | 4.54 | 5.49 | 6.62 | 8.13 | 10.73 | 13.29 |
| 2018 | 0.81 | 1.42 | 2.27 | 3.05 | 3.86 | 4.68 | 5.66 | 6.87 | 8.45 | 11.15 | 13.87 |
| 2019 | 0.83 | 1.43 | 2.30 | 3.12 | 3.96 | 4.82 | 5.83 | 7.06 | 8.71 | 11.53 | 14.53 |
| 2020 | 0.85 | 1.44 | 2.33 | 3.19 | 4.02 | 4.91 | 6.00 | 7.23 | 9.02 | 11.91 | 15.29 |
| 2021 | 0.83 | 1.48 | 2.37 | 3.26 | 4.13 | 5.07 | 6.16 | 7.44 | 9.26 | 12.35 | 15.86 |
| 2022 | 0.84 | 1.49 | 2.44 | 3.33 | 4.22 | 5.17 | 6.30 | 7.63 | 9.58 | 12.94 | 16.33 |
| 2023 | 0.85 | 1.52 | 2.50 | 3.40 | 4.30 | 5.32 | 6.47 | 7.85 | 9.79 | 13.40 | 16.60 |
| 2024 | 0.87 | 1.55 | 2.57 | 3.46 | 4.39 | 5.44 | 6.61 | 8.06 | 10.12 | 13.61 | 17.06 |
| 2025 | 0.89 | 1.59 | 2.62 | 3.52 | 4.53 | 5.58 | 6.74 | 8.28 | 10.44 | 14.02 | 17.54 |
| 2026 | 0.92 | 1.62 | 2.67 | 3.62 | 4.59 | 5.69 | 6.89 | 8.52 | 10.70 | 14.36 | 17.85 |
| 2027 | 0.91 | 1.64 | 2.71 | 3.70 | 4.69 | 5.79 | 7.04 | 8.65 | 10.94 | 14.72 | 18.29 |
| 2028 | 0.93 | 1.68 | 2.78 | 3.76 | 4.77 | 5.93 | 7.15 | 8.80 | 11.18 | 15.01 | 18.95 |
| 2029 | 0.94 | 1.71 | 2.83 | 3.83 | 4.89 | 6.04 | 7.32 | 9.01 | 11.38 | 15.29 | 19.31 |
| 2030 | 0.97 | 1.75 | 2.88 | 3.92 | 4.97 | 6.11 | 7.47 | 9.17 | 11.58 | 15.63 | 19.58 |
| **LCEF Real Maturity Value** | $ 73.86 | $ 89.07 | $ 108.73 | $ 125.05 | $ 140.00 | $ 156.35 | $ 174.99 | $ 196.21 | $ 224.16 | $ 269.53 | $ 311.97 |

# Monte Carlo Simulation Analysis of Managed Endowment Spending Program (MESP)

Asset Allocation

| Wilshire | Lehman | MSCIxUS | TIPS | Cash | REIT |
|---|---|---|---|---|---|
| 56.00% | 16.00% | 12.00% | 11.00% | 1.00% | 4.00% |

**Input Region:**

| | |
|---|---|
| Real Expected Return: | 7.5% |
| Standard Deviation | 20.0% |
| Terminal Horizon: | 30 |
| Shortfall Tolerance: | 18% |
| Lambda [0,1]: | 0.2 |
| Initial Spending: | $4.32 |
| Initial Wealth | $100 |
| Current Value | $100 |

$$s_i = (1-\lambda)s_{i-1} + \lambda V_i \max[GM - \frac{Ln(100/V_i)+K}{T}, 0]$$

| | |
|---|---|
| Real Geometric Mean: | 5.50% |
| Prudence (K) | 1.00273 |

| | |
|---|---|
| **Prob. of Maintaining $100 (after inflation) at 30-year Horizon** | 82.31% |

**Output in Real Value**

*The percentile value of actual real spending -- in each year -- based on the given rule.*

| Year | 5% | 10% | 20% | 30% | 40% | 50% | 60% | 70% | 80% | 90% | 95% |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2001 | 3.75 | 3.78 | 3.83 | 3.86 | 3.89 | 3.92 | 3.95 | 3.98 | 4.02 | 4.09 | 4.14 |
| 2002 | 3.26 | 3.33 | 3.42 | 3.48 | 3.55 | 3.61 | 3.68 | 3.75 | 3.84 | 3.97 | 4.09 |
| 2003 | 2.84 | 2.95 | 3.08 | 3.18 | 3.27 | 3.38 | 3.49 | 3.61 | 3.76 | 3.97 | 4.18 |
| 2004 | 2.49 | 2.62 | 2.79 | 2.93 | 3.07 | 3.21 | 3.36 | 3.53 | 3.74 | 4.07 | 4.36 |
| 2005 | 2.17 | 2.34 | 2.56 | 2.74 | 2.92 | 3.09 | 3.29 | 3.53 | 3.80 | 4.23 | 4.63 |
| 2006 | 1.91 | 2.10 | 2.38 | 2.61 | 2.81 | 3.03 | 3.27 | 3.56 | 3.92 | 4.46 | 5.02 |
| 2007 | 1.67 | 1.90 | 2.24 | 2.50 | 2.76 | 3.01 | 3.30 | 3.63 | 4.08 | 4.77 | 5.45 |
| 2008 | 1.49 | 1.75 | 2.13 | 2.44 | 2.74 | 3.03 | 3.37 | 3.76 | 4.26 | 5.11 | 5.92 |
| 2009 | 1.34 | 1.62 | 2.05 | 2.41 | 2.75 | 3.09 | 3.45 | 3.93 | 4.50 | 5.53 | 6.41 |
| 2010 | 1.21 | 1.51 | 1.99 | 2.39 | 2.77 | 3.16 | 3.58 | 4.12 | 4.78 | 5.93 | 6.94 |
| 2011 | 1.11 | 1.44 | 1.96 | 2.39 | 2.81 | 3.24 | 3.72 | 4.32 | 5.10 | 6.38 | 7.60 |
| 2012 | 1.00 | 1.38 | 1.93 | 2.41 | 2.87 | 3.35 | 3.90 | 4.54 | 5.41 | 6.81 | 8.23 |
| 2013 | 0.93 | 1.32 | 1.92 | 2.45 | 2.94 | 3.46 | 4.06 | 4.80 | 5.73 | 7.34 | 8.92 |
| 2014 | 0.88 | 1.28 | 1.92 | 2.49 | 3.02 | 3.58 | 4.25 | 5.05 | 6.08 | 7.87 | 9.62 |
| 2015 | 0.83 | 1.24 | 1.94 | 2.54 | 3.11 | 3.73 | 4.43 | 5.30 | 6.48 | 8.39 | 10.36 |
| 2016 | 0.79 | 1.24 | 1.96 | 2.59 | 3.22 | 3.87 | 4.63 | 5.59 | 6.84 | 8.95 | 11.02 |
| 2017 | 0.77 | 1.24 | 1.97 | 2.66 | 3.31 | 4.00 | 4.84 | 5.86 | 7.23 | 9.57 | 11.88 |
| 2018 | 0.74 | 1.25 | 2.00 | 2.73 | 3.41 | 4.18 | 5.06 | 6.12 | 7.57 | 10.15 | 12.61 |
| 2019 | 0.75 | 1.26 | 2.06 | 2.81 | 3.51 | 4.34 | 5.27 | 6.39 | 7.92 | 10.57 | 13.42 |
| 2020 | 0.74 | 1.26 | 2.10 | 2.87 | 3.62 | 4.49 | 5.46 | 6.66 | 8.28 | 11.13 | 14.34 |
| 2021 | 0.75 | 1.28 | 2.14 | 2.93 | 3.74 | 4.62 | 5.67 | 6.91 | 8.68 | 11.67 | 15.06 |
| 2022 | 0.75 | 1.29 | 2.19 | 3.02 | 3.85 | 4.78 | 5.85 | 7.19 | 9.01 | 12.30 | 15.73 |
| 2023 | 0.75 | 1.34 | 2.24 | 3.11 | 3.96 | 4.92 | 6.04 | 7.47 | 9.33 | 12.91 | 16.49 |
| 2024 | 0.76 | 1.35 | 2.30 | 3.17 | 4.08 | 5.08 | 6.28 | 7.70 | 9.71 | 13.44 | 17.30 |
| 2025 | 0.75 | 1.40 | 2.37 | 3.27 | 4.19 | 5.22 | 6.47 | 7.96 | 10.06 | 13.96 | 17.79 |
| 2026 | 0.78 | 1.43 | 2.42 | 3.33 | 4.32 | 5.38 | 6.62 | 8.23 | 10.40 | 14.44 | 18.35 |
| 2027 | 0.81 | 1.46 | 2.47 | 3.42 | 4.41 | 5.53 | 6.83 | 8.47 | 10.80 | 14.85 | 18.98 |
| 2028 | 0.81 | 1.49 | 2.54 | 3.51 | 4.52 | 5.67 | 6.98 | 8.66 | 11.18 | 15.31 | 19.53 |
| 2029 | 0.84 | 1.52 | 2.59 | 3.59 | 4.65 | 5.83 | 7.19 | 8.92 | 11.46 | 15.71 | 20.22 |
| 2030 | 0.85 | 1.54 | 2.67 | 3.68 | 4.77 | 5.98 | 7.35 | 9.14 | 11.75 | 16.28 | 20.86 |
| **LCEF Real Maturity Value** | $ 67.22 | $ 83.15 | $ 104.38 | $ 122.33 | $ 139.51 | $ 157.75 | $ 179.57 | $ 204.95 | $ 238.90 | $ 296.34 | $ 351.52 |